UK INTERMEDIATE MATHEMATICAL CHALLENGE

THURSDAY 3RD FEBRUARY 2011

Organised by the **United Kingdom Mathematics Trust**
and supported by

The Actuarial Profession
making financial sense of the future

RULES AND GUIDELINES (to be read before starting)

1. Do not open the paper until the Invigilator tells you to do so.

2. Time allowed: **1 hour**.
    No answers, or personal details, may be entered after the allowed hour is over.

3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.

4. Candidates in England and Wales must be in School Year 11 or below.
    Candidates in Scotland must be in S4 or below.
    Candidates in Northern Ireland must be in School Year 12 or below.

5. **Use B or HB pencil only**. Mark *at most one* of the options A, B, C, D, E on the
    Answer Sheet for each question. Do not mark more than one option.

6. *Do not expect to finish the whole paper in 1 hour.* Concentrate first on Questions 1-15.
    When you have checked your answers to these, have a go at some of the later questions.

7. Five marks are awarded for each correct answer to Questions 1-15.
    Six marks are awarded for each correct answer to Questions 16-25.
    Each incorrect answer to Questions 16-20 loses 1 mark.
    Each incorrect answer to Questions 21-25 loses 2 marks.

8. Your Answer Sheet will be read only by a *dumb machine*. **Do not write or doodle on the
    sheet except to mark your chosen options**. The machine 'sees' all black pencil markings
    even if they are in the wrong places. If you mark the sheet in the wrong place, or leave bits of
    rubber stuck to the page, the machine will 'see' a mark and interpret this mark in its own way.

9. The questions on this paper challenge you to **think**, not to guess. You get more marks,
    and more satisfaction, by doing one question carefully than by guessing lots of answers.
    The UK IMC is about solving interesting problems, not about lucky guessing.

1. What is the value of 4.5 × 5.5 + 4.5 × 4.5?

   A 36.5   B 45   C 50   D 90   E 100

2. To find the diameter in mm of a Japanese knitting needle, you multiply the size by 0.3 and add 2.1. What is the diameter in mm of a size 5 Japanese knitting needle?

   A 3.6   B 7.4   C 10.8   D 12   E 17.1

3. The consecutive digits 1, 2, 3, 4 in that order can be arranged to make the correct division, $12 \div 3 = 4$. One *other* sequence of four consecutive digits p, q, r, s makes a correct division, 'pq' $\div r = s$. What is the value of s in this case?

   A 4   B 5   C 6   D 7   E 8

4. The angles of a triangle are in the ratio 2:3:5. What is the difference between the largest angle and the smallest angle?

   A 9°   B 18°   C 36°   D 45°   E 54°

5. The diagram shows a rectangle placed on a grid of 1 cm×1 cm squares. What is the area of the rectangle in cm²?

   A 15   B $22\frac{1}{2}$ C 30   D 36   E 45
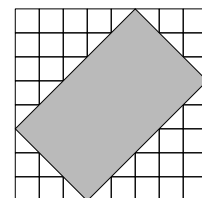
6. When I glanced at my car milometer it showed 24942, a palindromic number. Two days later, I noticed that it showed the next palindromic number. How many miles did my car travel in those two days?

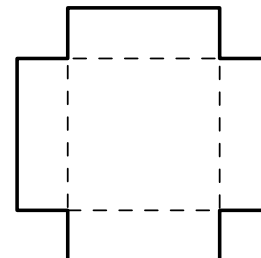   A 100   B 110   C 200   D 220   E 1010

7. What is the value of x in this diagram?

   A 30   B 35   C 40   D 45   E 50
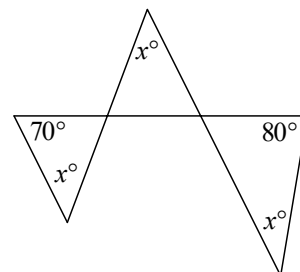
8. A square piece of card has a square of side 2 cm cut out from each of its corners. The remaining card is then folded along the dotted lines shown to form an open box whose total internal surface area is 180 cm².
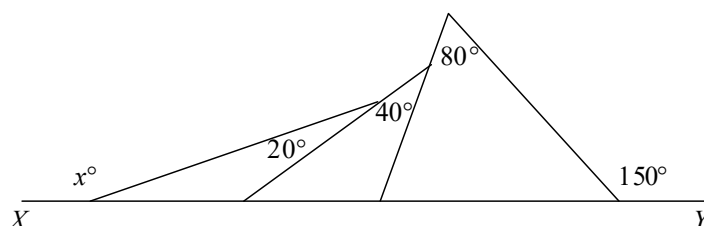
   What is the volume of the open box in cm³?

   A 100   B 128   C 162   D 180   E 200

9. In the diagram, XY is a straight line. What is the value of x?

   A 170   B 160   C 150   D 140   E 130

10. Merlin magically transforms a 6 tonne monster into mice with the same total mass. Each mouse has a mass of 20g. How many mice does Merlin make?

   A 30 B 300 C 3000 D 30 000 E 300 000

11. What is the value of $19\frac{1}{2} \times 20\frac{1}{2}$?

   A 250 B $380\frac{1}{4}$ C $390\frac{1}{4}$ D 395 E $399\frac{3}{4}$

12. What is the sum of the first 2011 digits when $20 \div 11$ is written as a decimal?

   A 6013 B 7024 C 8035 D 9046 E 10057

13. The three blind mice stole a piece of cheese. In the night, the first mouse ate $\frac{1}{3}$ of the cheese. Later, the second mouse ate $\frac{1}{3}$ of the remaining cheese. Finally, the third mouse ate $\frac{1}{3}$ of what was then left of the cheese.
   Between them, what fraction of the cheese did they eat?

   A $\dfrac{16}{27}$ B $\dfrac{17}{27}$ C $\dfrac{2}{3}$ D $\dfrac{19}{27}$ E $\dfrac{20}{27}$

14. The number 6 lies exactly halfway between 3 and 3^2. Which of the following is not halfway between a positive integer and its square?

   A 3 B 10 C 15 D 21 E 30

15. The equilateral triangle ABC has sides of length 1 and AB lies on the line XY. The triangle is rotated clockwise around B until BC lies on the line XY. It is then rotated similarly around C and then about A as shown in the diagram.
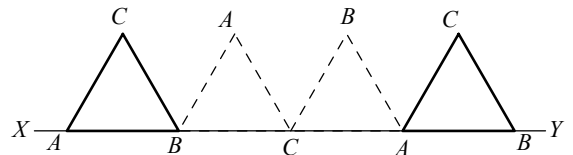

   What is the length of the path traced out by point C during this sequence of rotations?

   A $\dfrac{4\pi}{3}$ B $2\sqrt{3}$ C $\dfrac{8\pi}{3}$ D 3 E $\dfrac{2\pi}{3}$

   _____

16. The diagram shows an L-shape divided into 1×1 squares. Gwyn cuts the shape along some of the lines shown to make two pieces, neither of which is a square. She then uses the pieces to form a 2×6 rectangle.
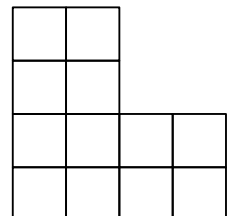
   What is the difference between the areas of the two pieces?

   A 0 B 1 C 2 D 3 E 4

17. A shop advertised "Everything half price in our sale", but also now advertises that there is "An additional 15% off sale prices". Overall, this is equivalent to what reduction on the original prices?

   A 7.5% B 35% C 57.5% D 65% E 80%

18. The diagram contains six equilateral triangles with sides of length 2 and a regular hexagon with sides of length 1.
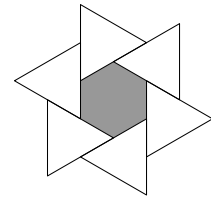
What fraction of the whole shape is shaded?

A $\dfrac{1}{8}$ B $\dfrac{1}{7}$ C $\dfrac{1}{6}$ D $\dfrac{1}{5}$ E $\dfrac{1}{4}$

19. Harrogate is 23km due north of Leeds, York is 30km due east of Harrogate, Doncaster is 48km due south of York, and Manchester is 70km due west of Doncaster. To the nearest kilometre, how far is it from Leeds to Manchester, as the crow flies?

A 38km   B 47km   C 56km   D 65km   E 74km

20. Max and his dog Molly set out for a walk. Max walked up the road and then back down again, completing a six mile round trip. Molly, being an old dog, walked at half Max's speed. When Max reached the end of the road, he turned around and walked back to the starting point, at his original speed. Part way back he met Molly, who then turned around and followed Max home, still maintaining her original speed. How far did Molly walk?

A 1 mile   B 2 miles   C 3 miles   D 4 miles   E 5 miles

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21. A regular octagon is placed inside a square, as shown. The shaded square connects the midpoints of four sides of the octagon.
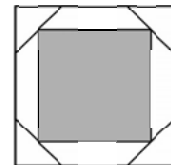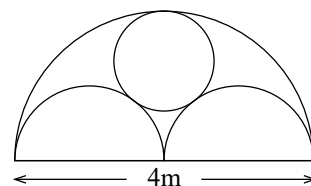
What fraction of the outer square is shaded?

A $\sqrt{2} - 1$ B $\dfrac{1}{2}$ C $\dfrac{\sqrt{2}+1}{4}$ D $\dfrac{\sqrt{2}+2}{5}$ E $\dfrac{3}{4}$

22. You are given that $5^p = 9$, $9^q = 12$, $12^r = 16$, $16^s = 20$ and $20^t = 25$. What is the value of $pqrst$?

A 1   B 2   C 3   D 4   E 5

23. A window frame in Salt's Mill consists of two equal semicircles and a circle inside a large semicircle with each touching the other three as shown. The width of the frame is 4m.

What is the radius of the circle, in metres?

A $\dfrac{2}{3}$ B $\dfrac{\sqrt{2}}{2}$ C $\dfrac{3}{4}$ D $2\sqrt{2} - 1$ E 1

24. Given any positive integer n, Paul adds together the distinct factors of n, other than n itself. Which of these numbers can never be Paul's answer?

A 1   B 3   C 5   D 7   E 9

25. The diagram shows a square, a diagonal and a line joining a vertex to the midpoint of a side. What is the ratio of area P to area Q?

A $1 : \sqrt{2}$ B $2 : 3$ C $1 : 2$ D $2 : 5$ E $1 : 3$